647510 **BSCmaMJ206x-3C** **Seat No :_____**

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2024
MATH: Advanced Calculus -1 (Major-6)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two. **(10)**

(1) Define: Curl of a vector function. Prove that
$$curl(\varphi\bar{f}) = \varphi curl\bar{f} + (grad\varphi) \times \bar{f}.$$

(2) Define: Irrotational function. If $\bar{f}$ is irrotational then find a, b, c, where $\bar{f} = (2x + 3y + az)\hat{\imath} + (bx + 2y + 3z)\hat{\jmath} + (2x + cy + 3z)\hat{k}$.

(3) If $r = |\bar{r}|$, where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then prove that

(i) $\nabla r^n = nr^{n-2}\bar{r}$ (ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

Que.2 Answer any two. **(10)**

(1) Evaluate $\iint_R xcos(x + y)dxdy$, where R is a triangle with vertices $(0,0), (\pi, 0)$ and (π, π).

(2) Prove that: $\iint_R \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}dxdy = \frac{\pi}{4}\left(\frac{\pi}{2} - 1\right)$, where R is the first quadrant of unit circle.

(3) Evaluate: $\iiint_R \frac{dxdydz}{(x+y+z+1)^3}$; where R is bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

Que.3 Answer any two. **(10)**

(1) Evaluate: $\oint_C \bar{F} \cdot d\bar{r}$; where $\bar{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ and c is a rectangle obtained by $y = 0, x = a, y = b$ and $x = 0$.

(2) Find $\iint_S (xcos\alpha + ycos\beta + zcos\gamma)d\sigma$, where s is a hemisphere $z = \sqrt{1 - x^2 - y^2}$.

(3) Verify Stoke's theorem for $\bar{f} = (2x - y, -yz^2, -y^2z)$ and s is upper half of unit sphere.

Que.4 Answer any two. **(10)**

(1) Define: Beta function. Prove that : $\beta(p, q) = \int_0^{\propto} \frac{y^{p-1}}{(1+y)^{p+q}}dy$; $p > 0, q > 0$.

(2) Prove: $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$; $p > 0, q > 0$.

(3) Prove: (i) $p\beta(p, q + 1) = q\beta(p + 1, q); p > 0, q > 0$.

(ii) $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{sin\theta}}d\theta \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}d\theta = \pi$.

Que.5 Answer any two. **(10)**

(1) Define: Solenoidal function. If $\bar{f}$ is a vector function defined on D, whose second order derivatives exists then prove that its curl is solenoidal.

(2) State: Green's theorem. Evaluate: $\oint_c (y - sinx)dx + cosxdy$; where c is a triangle formed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.

(3) Define: Gamma function. Prove that: $\Gamma(p) = 2\int_0^{\propto} e^{-x^2}x^{2p-1}\,dx;\ p > 0$.

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